

1-6

Compound Inequalities

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I CAN... write and solve compound inequalities.

VOCABULARY

- compound inequality

EXPLORE & REASON

Hana has some blue paint. She wants to lighten the shade, so she mixes in 1 cup of white paint. The color is still too dark, so Hana keeps mixing in 1 cup of white paint at a time. After adding 4 cups, she decides the color is too light.



- Explain in words how much paint Hana should have added initially to get the shade she wants.
- Model With Mathematics** Represent your answer to part A with one or more inequalities.
- Hana decides that she likes the shades of blue that appear in between adding 1 cup and 4 cups of white paint. How can you represent the number of cups of white paint that yield the shades Hana prefers?

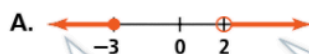
ESSENTIAL QUESTION

CONCEPTUAL UNDERSTANDING

What are compound inequalities and how are their solutions represented?

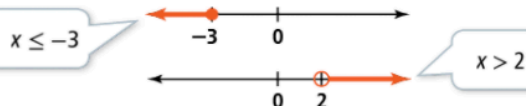
EXAMPLE 1 Understand Compound Inequalities

How can you use inequalities to describe the sets of numbers graphed below?



The graph shows the solutions of two inequalities. The two inequalities form a *compound inequality*. A **compound inequality** is made up of two or more inequalities.

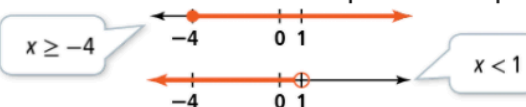
Write an inequality to represent the solutions shown in each part of the graph.



The compound inequality that describes the graph is $x \leq -3$ or $x > 2$.



The solutions shown in the graph are greater than or equal to -4 . They are also less than 1 . Write two inequalities to represent this.



The compound inequality that describes the graph is $-4 \leq x$ and $x < 1$. You can also write this as $-4 \leq x < 1$.

MAKE SENSE AND PERSEVERE

There is no number that can be less than -3 AND greater than 2 . So it makes sense to use OR to write the compound inequality.

Try It! 1. Write a compound inequality for the graph.





Activity



Assess

**EXAMPLE 2** Solve a Compound Inequality Involving *Or*

Solve the compound inequality $5x - 7 < 13$ or $-4x + 3 > 11$. Graph the solution.

Solve each inequality.

$$5x - 7 < 13 \quad \text{or} \quad -4x + 3 > 11$$

$$5x - 7 + 7 < 13 + 7 \quad -4x + 3 - 3 > 11 - 3$$

$$5x < 20$$

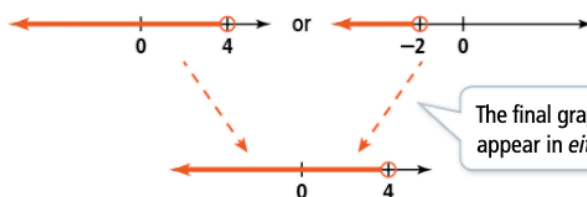
$$-4x > 8$$

$$\frac{5x}{5} < \frac{20}{5}$$

$$\frac{-4x}{-4} < \frac{8}{-4}$$

$$x < 4$$

$$x < -2$$



The solution is $x < 4$, which is the set of all real numbers less than 4.



Try It! 2. Solve the compound inequality $-3x + 2 > -7$ or $2(x - 2) \geq 6$. Graph the solution.

**EXAMPLE 3** Solve a Compound Inequality Involving *And*

What is the solution of $-12 \leq 7x + 9 < 16$?

Solve each inequality.

$$-12 \leq 7x + 9 \quad \text{and} \quad 7x + 9 < 16$$

$$-12 - 9 \leq 7x + 9 - 9 \quad 7x + 9 - 9 < 16 - 9$$

$$-21 \leq 7x$$

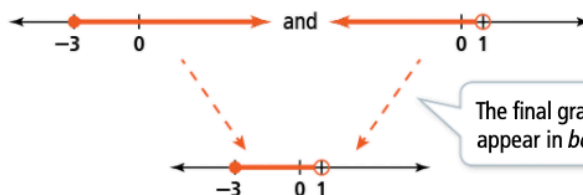
$$7x < 7$$

$$\frac{-21}{7} \leq \frac{7x}{7}$$

$$\frac{7x}{7} < \frac{7}{7}$$

$$-3 \leq x$$

$$x < 1$$



The solution is $x \geq -3$ and $x < 1$, or $-3 \leq x < 1$.



Try It! 3. Solve the compound inequality $-2(x + 1) < 4$ and $4x + 1 \leq -3$. Graph the solution.

COMMON ERROR

You may think that there should be two parts in the graph of the solutions. However, the solution to $x < -2$ is a subset of the solution to $x < 4$, so $x < 4$ is the complete solution.





APPLICATION



EXAMPLE 4

Solve Problems Involving Compound Inequalities

Enrique plans a diet for his dog, River. River consumes between 510 and 540 Calories per day.

If River eats $1\frac{1}{2}$ servings of dog food each day, how many treats can she have?



Formulate

Model the situation with a compound inequality.

Let x represent the number of treats River can have each day.

Write an expression to represent River's total daily Calories.

$1\frac{1}{2}$ servings at 320 Cal. per serving plus x treats at 15 Cal. per treat

480

+

15x

Write a compound inequality for the number of dog treats each day.

at least 510 Calories

at most 540 Calories

$$510 \leq 480 + 15x \leq 540$$

Compute

Solve the compound inequality.

$$510 \leq 480 + 15x \leq 540$$

$$510 - 480 \leq 480 + 15x - 480 \leq 540 - 480$$

$$30 \leq 15x \leq 60$$

$$\frac{30}{15} \leq \frac{15x}{15} \leq \frac{60}{15}$$

$$2 \leq x \leq 4$$

The solution is $2 \leq x \leq 4$.

Interpret

River can have at least 2 and at most 4 treats each day.



Try It!

4. Suppose River has new treats that are 10 Calories each. How many of the new treats can she have and remain in her Calorie range?



CONCEPT SUMMARY Compound Inequalities



Concept
Summary



Assess

WORDS

The solution of a compound inequality involving **or** includes the solutions of one inequality as well as the solutions of the other inequality.

The solution of a compound inequality involving **and** includes only solutions of both inequalities where they coincide.

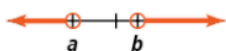
ALGEBRA

$$x < a \text{ or } x > b$$

$$x > a \text{ and } x < b$$

$$a < x < b$$

GRAPHS



Do You UNDERSTAND?

- ESSENTIAL QUESTION** What are compound inequalities and how are their solutions represented?
- Look for Relationships** When $a < b$, how is the graph of $x > a$ and $x < b$ similar to the graph of $x > a$? How is it different?
- Vocabulary** A *compound* is defined as a *mixture*. Make a conjecture as to why the term *compound inequality* includes the word *compound*.
- Error Analysis** Kona graphed the compound inequality $x > 2$ or $x > 3$ by graphing $x > 3$. Explain Kona's error.

Do You KNOW HOW?

Write a compound inequality for each graph.



Solve each compound inequality and graph the solution.

7. $4x - 1 > 3$ and $-2(3x - 4) \geq -16$

8. $2(4x + 3) \geq -10$ or $-5x - 15 > 5$

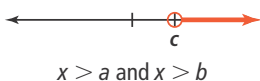
9. Nadeem plans to ride her bike between 12 mi and 15 mi. Write and solve an inequality to model how many hours Nadeem will be riding.



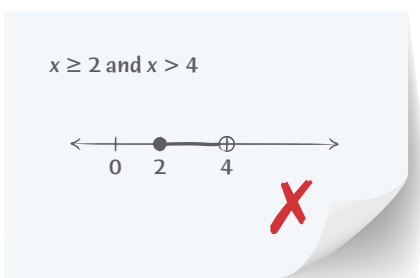


UNDERSTAND

10. **Look for Relationships** The compound inequality $x > a$ and $x > b$ is graphed below. How is the point labeled c related to a and b ?



11. **Error Analysis** Describe and correct the error a student made graphing the compound inequality $x \geq 2$ and $x > 4$.



12. **Generalize** Suppose that $a < b$. Select from the symbols $>$, $<$, \geq , and \leq , as well as the words *and* and *or*, to complete the compound inequality below so that its solution is all real numbers.

$$x \square a \square x \square b$$

13. **Higher Order Thinking** Let a and b be real numbers.
- If $a > b$, how is the graph of $x > a$ and $x > b$ different from the graph of $x > a$ or $x > b$?
 - If $a < b$, how is the graph of $x > a$ and $x > b$ different from the graph of $x > a$ or $x > b$?
 - If $a = b$, how is the graph of $x > a$ and $x > b$ different from the graph of $x > a$ or $x > b$?

14. **Mathematical Connections** Consider the solutions of the compound inequalities.

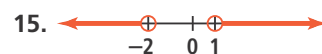
$$4 < x < 8 \quad 2 < x < 11$$

Describe each solution as a set. Is one set a subset of the other? Explain your answer.

PRACTICE

Write a compound inequality for each graph.

SEE EXAMPLE 1

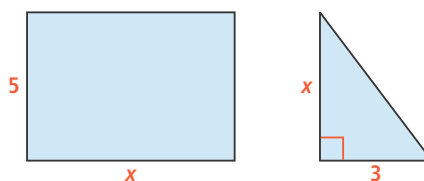


Solve each compound inequality and graph the solution. SEE EXAMPLES 2 AND 3

- $2x + 5 > -3$ and $4x + 7 < 15$
- $2x - 5 > 3$ or $-4x + 7 < -25$
- $2x - 5 > 3$ and $-4x + 7 < -25$
- $-x + 1 > -2$ or $6(2x - 3) \geq -6$
- $-x + 1 > -2$ and $6(2x - 3) \geq -6$
- $-\frac{5}{8}x + 2 + \frac{3}{4}x > -1$ or $-3(x + 25) > 15$

The value for the area A of each figure is given. Write and solve a compound inequality for the value of x in each figure. SEE EXAMPLE 4

25. $35 \geq A \geq 25$ 26. $9 \leq A \leq 12$



Write a compound inequality to represent each sentence below. SEE EXAMPLE 4

- A quantity x is at least 10 and at most 20.
- A quantity x is either less than 10 or greater than 20.
- A quantity x is greater than 10 and less than 20.

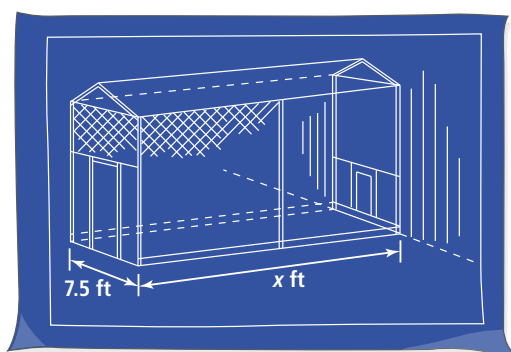


APPLY

30. **Reason** Fatima plans to spend at least \$15 and at most \$20 on sketch pads and pencils. If she buys 2 sketch pads, how many pencils can she buy while staying in her price range?



31. **Make Sense and Persevere** A peanut company ships its product in a carton that weighs 20 oz when empty. Twenty bags of peanuts are shipped in each carton. The acceptable weight for one bag of peanuts is between 30.5 oz and 33.5 oz, inclusive. If a carton weighs too much or too little, it is opened for inspection. Write and solve a compound inequality to determine x , the weights of cartons that are opened for inspection.
32. **Model With Mathematics** Volunteers at an animal shelter are building a rectangular dog run so that one shorter side of the rectangle is formed by the shelter building as shown. They plan to spend between \$100 and \$200 on fencing for the sides at a cost of \$2.50 per ft. Write and solve a compound inequality to model the possible length of the dog run.



ASSESSMENT PRACTICE

33. Which of the following compound inequalities have the solution $x < 3$? Select all that apply.

- (A) $3x + 5 < 6$ or $-2x + 9 > 3$
- (B) $3x + 5 < 6$ and $-2x + 9 > 3$
- (C) $3x - 5 < 10$ and $-2x + 9 > 3$
- (D) $3x + 5 < 6$ or $-2x + 9 < 3$
- (E) $3x - 5 < 10$ or $-2x + 9 > 3$

34. **SAT/ACT** What is the solution of $0.2x - 4 - 2x < -0.4$ and $3x + 2.7 < 3$?

- (A) $x < -2$
- (B) $x < 0.1$
- (C) $x < 1$
- (D) $x > -2$ and $x < 0.1$
- (E) $x > -2$ and $x < 1$

35. **Performance Task** An animal shelter categorizes donors based on their total yearly donation, as shown in the table.



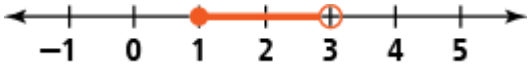
Donor Category	Total Yearly Donation
Bronze	$< \$100$
Silver	$\geq \$100$ and $< \$500$
Gold	$\geq \$500$ and $< \$1,000$
Platinum	$\geq \$1,000$

Part A Keenan donates the same amount each month. Write and solve a compound inequality for the monthly donation that will put him in the Gold category.

Part B Libby donated \$50 during the first month of the year. If she makes three additional donations of equal amounts during the year, how much will she need to donate each time to be in the Silver category?

Part C Paula originally planned to donate \$50 each month. After reviewing her budget, she decides that she must reduce her planned donation. By what amount can she reduce her original planned monthly donation of \$50 so that she will be in the Silver category?



11. The student graphed $x \geq 2$ and $x < 4$. The student should have graphed an open circle on 4 with the arrow pointing right.
13. a. The graph of $x > a$ and $x > b$ has an open circle on a with the arrow pointing right, *while* the graph of $x > a$ or $x > b$ has an open circle on b with the arrow pointing right.
- b. The graph of $x > a$ and $x > b$ has an open circle on b with the arrow pointing right, while the graph of $x > a$ or $x > b$ has an open circle on a with the arrow pointing right.
- c. The graphs are not different; they are the same.
15. $x < -2$ or $x > 1$
17. $x \geq -0.5$ and $x < 0.25$, or $-0.5 \leq x < 0.25$
19. $x > -4$ and $x < 2$;
- 
21. $x > 8$;
- 
23. $1 \leq x < 3$;
- 
25. $7 \geq x \geq 5$, or $5 \leq x \leq 7$
27. $x \geq 10$ and $x \leq 20$, or $10 \leq x \leq 20$
29. $x > 10$ and $x < 20$, or $10 < x < 20$
31. Expressions may vary. Sample: $x < 20 + 20(30.5)$ or $x > 20 + 20(33.5)$; $x < 630$ or $x > 690$; Cartons less than 630 ounces and greater than 690 ounces should be opened for inspection.
33. A, C

- 35. Part A** $500 \leq 12x < 1000$; $41.\bar{6} \leq x < 83.\bar{3}$; A monthly donation from \$41.67 to \$83.33 will put Keenan in the Gold category.
- Part B** $100 \leq 50 + 3x < 500$; $16.\bar{6} \leq x < 150$; Three donations from \$16.67 to \$150 will put Libby in the Silver category.
- Part C** $100 \leq 12(50 - x) < 500$; $8.\bar{3} < x \leq 41.\bar{6}$; Paula can reduce her monthly contribution by \$8.34 to \$41.66 dollars per month.